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Comments on "Remarks on statistical errors in equivalent width" by K. Vollmann & T. Eversberg

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Both Chalabaev & Maillard and Vollmann & Eversberg proposed calculation methods for the statistical error of the equivalent width. Both methods can lead to very different error bars. Here we present a straightforward expression for normalized spectra, which makes use of only standard commands of IRAF and favors the first method.

KEYWORDS
line: profiles – methods: statistical – techniques: spectroscopic

1 | INTRODUCTION

Vollmann & Eversberg (2006) wrote a paper where they questioned results of Chalabaev & Maillard (1983) based on the calculation of the statistical errors of the equivalent width. The method of calculation is described in the appendix to their paper. Vollmann & Eversberg use for their method of calculation some estimated mean values of flux quantities and arrive at significantly higher values of the statistical errors than those obtained by Chalabaev & Maillard, which led them to the conclusion that in the Chalabaev & Maillard paper a proposed correlation of H_α and H_β is not valid.

This paper presents a more straightforward approach for the calculation of the statistical error of an equivalent width based on the Gaussian law of statistical error propagation and a simplification of the calculation without the estimated mean values. Systematic errors are not considered.

2 | THE ERROR OF EQUIVALENT WIDTH

The equivalent width of a line at wavelength λ_0 is defined by

$$W = \int_{\lambda_1}^{\lambda_2} \left(1 - \frac{F(\lambda)}{Fc(\lambda)}\right) d\lambda, \tag{1}$$

where $F(\lambda)$ and $F_c(\lambda)$ are the measured flux of the spectrum and of the continuum at wavelength λ, respectively. Let the line stretch over a series of pixels numbered from 0 to n. In practice, we have for each pixel $0 \le \nu \le n$ the measured observables F_ν and the quantities Fc_ν for the flux and continuum. The latter is usually for non-normalized spectra defined as a polynomial fit of the flux F_{c_μ} of, say m $(1 \le \mu \le m)$ the pixel of the continuum in the vicinity of the line. Let their standard derivation be σ_{Fc_μ}, and the standard derivation of Fc_ν is (Barlow 1988)

$$\sigma_{Fc_\nu} \sim \frac{1}{\sqrt{m}} \sigma_{Fc_\mu} \le \sigma_{Fc_\mu}.$$

Each pixel covers a separate wavelength interval ν with the flux F_ν, which means that $F(\lambda)$ and $Fc(\lambda)$ are step functions. Therefore, we replace the integral (1) by the sum

$$\begin{aligned} W(F) &= \Delta\lambda \sum_{\nu=0}^{n} \left(1 - \frac{F(\lambda)}{Fc(\lambda)}\right) \\ &= \lambda_n - \lambda_0 + \Delta\lambda \sum_{\nu=0}^{n} \frac{F(\lambda)}{Fc(\lambda)} \end{aligned} \tag{2}$$

with $\lambda_n > \lambda_0$ and the spectral dispersion per pixel $\Delta\lambda = \frac{\lambda_n - \lambda_0}{n}$. Each of the measurable quantities F_ν and Fc_ν is subject to measurement errors. According the Gaussian error propagation law, we get

$$\begin{aligned} {\sigma_W}^2 &= \sum_{\nu=0}^{n} \left[\left(\frac{\partial W}{\partial F_\nu}\right)^2 {\sigma_{F_\nu}}^2 + \left(\frac{\partial W}{\partial Fc_\nu}\right)^2 {\sigma_{Fc_\nu}}^2 \right] \\ &= \Delta\lambda^2 \sum_{\nu=0}^{n} \left[\left(\frac{1}{Fc_\nu}\right)^2 {\sigma_{F_\nu}}^2 + \left(\frac{F_\nu}{{Fc_\nu}^2}\right)^2 {\sigma_{Fc_\nu}}^2 \right]. \end{aligned} \tag{3}$$

Note the difference to (A3) and (A4) in Chalabaev & Maillard (1983). This approach assumes the statistical independence of the F_ν, which is of course not exactly the case. When the photon noise dominates, we can treat the noise as a stochastic Poisson process. We have

$$\sigma_{F_\nu} = \sqrt{\frac{F_\nu}{Fc_\nu}}\sigma_{Fc_\nu}. \tag{4}$$

Substituting Equation (4) into Equation (3) gives, under the assumption that σ_{Fc_λ} does not depend on λ

$$\sigma_W{}^2 = \Delta\lambda^2 \left(\sum_{\nu=0}^{n} \frac{F_\nu}{Fc_\nu{}^3} + \sum_{\nu=0}^{n} \frac{F_\nu{}^2}{Fc_\nu{}^4} \right) \sigma_{Fc}{}^2. \tag{5}$$

We specialize now Equation (5) for normalized spectra with $Fc_\nu = 1$.

$$\begin{aligned} \sigma_W^2 &= \Delta\lambda^2 \left(\sum_{\nu=0}^{n} F_\nu + \sum_{\nu=0}^{n} F_\nu{}^2 \right) \sigma_{Fc}^2 \\ &= \Delta\lambda(|\lambda_n - \lambda_0 - W(F)| + |\lambda_n - \lambda_0 - W(F^2)|)\sigma_{Fc}^2 \end{aligned} \tag{6}$$

using Equation (2) twice for the last equal sign. Note that $W(F^2)$ denotes the equivalent width of a spectrum where all flux and continuum values are replaced by their squares. Substituting $\frac{1}{S/N}$ for σ_{Fc}, we get our final result as it concerns the process of normalization:

$$\sigma_W = \frac{\sqrt{\Delta\lambda}}{S/N} \sqrt{|\ \lambda_n - \lambda_0 - W(F)\ | + |\ \lambda_n - \lambda_0 - W(F^2)\ |}. \tag{7}$$

At first glance, this formula looks unhandy because of the second term in the bracket on the right-hand side which contributes most. However, e.g. with the Iraf command sarith xyz.fit * xyz.fit sqxyz.fit, we can easily generate a spectrum with the squares of the intensities of the original spectrum. With splot xyz.fit resp. splot sqxyz.fit, we determine the equivalent widths of the original normalized spectrum and its square and the $\frac{S}{N}$ of the original spectrum.

At least we have to determine a basis that reflects the continuum of the normalized line. When we determine the line as the average of flux at the two endpoints $C = (Fc_{\lambda_n} + Fc_{\lambda_0})/2$, we have to replace Equation (2) with

$$W(F) = \Delta\lambda \sum_{\nu=0}^{n} \left(C - \frac{F(\lambda)}{Fc(\lambda)} \right) \tag{8}$$

which generates an additional term

$$\sigma_C = \frac{\lambda_n - \lambda_0}{\sqrt{2}^S/_N}. \tag{9}$$

Example: H_α from gam Cas token 2016/10/05 by the author: $\Delta\lambda = 0.0688\,\mathring{A}$, $\lambda_n - \lambda_0 = 65.5\ \mathring{A}$; $S/N = 222$; $W(F) = -40.3\ \mathring{A}$; $W\ (F^2) = -187.8\ \mathring{A}$; $\sigma_W = 0.0224\ \mathring{A}$; $\sigma_C = 0.209\,\mathring{A}$; $\sigma_{CW} = \sqrt{\sigma_W^2 + \sigma_C^2} = 0.210$ Å Vollmann & Eversberg (2006): <> stands for the average. Using the notation of this paper, eqs. (3) and (7) in Vollmann & Eversberg (2006) give

$$< F > = < Fc > \left(1 - \frac{W(F)}{\lambda_n - \lambda_0} \right) = 1.615,$$

$$\sigma_W = \sqrt{1 + \frac{< Fc >}{< F >}} \frac{\lambda_n - \lambda_0 - W(F)}{S/_N} = 0.61\,\mathring{A}.$$

Chalabaev & Maillard (1983): $\sigma_W = 0.23\,\mathring{A}$.

3 | SUMMARY AND CONCLUSIONS

Calculation of the statistical errors in equivalent widths according the method proposed by Vollmann & Eversberg may result in a highly overestimated error bars. The error bars calculated according Chalabaev & Maillard are regarded as more adequate.

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REFERENCES

Barlow, R. J. 1988, *Statistics*, Wiley (Chichester).
Chalabaev, A., & Maillard, J. P. 1983, *A&A*, *127*, 279.
Vollmann, K., & Eversberg, T. 2006, *AN*, *327*, 862.

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